Supplementary Information for "Forces and Transport Velocities for a Particle in a Slotted Waveguide" by Yang *et al.*

I. Maxwell Stress Tensor method

Validation of the finite element Maxwell Stress Tensor method was accomplished by creating a simulation that would mimic the conditions of a uniform plane wave incident on a nanoparticle. The simulation domain consists of a cube containing liquid water, with a dielectric nanoparticle suspended in the center of the cube. Surrounding the box region are PML boundaries designed to absorb all scattered and incident light. Using this geometry, we simulated the effects of a plane wave as it scatters off the nanoparticle, and the scattering force on the nanoparticle. Shown in Figure 1, we plot the results of these simulations against calculated results using the Rayleigh scattering equation, changing only the particle diameter in subsequent iterations. As expected, we note that there is an insignificant amount of force generated in directions orthogonal to optical propagation. However, for particles above 100 nm in diameter, deviations appear in simulated values compared to Rayleigh theory. Shown in Figure 1(b), plotting the convergence between simulated and calculated values versus the Rayleigh criterion ($D\pi/\lambda << 1$), where D is the particle diameter, and λ is the freespace wavelength, we see that as $D\pi/\lambda$ gets larger, convergence between the two methods decreases. This agrees with similar calculations conducted for particle propulsion on silicon nitride waveguides and previous work that observed size dependences much lower than the predicted 6th order and 3rd order dependencies for Rayleigh scattering and gradient forces. We see that for particles with diameters smaller than 100 nm, which are generally used in slot waveguides, the calculated MST value shows less than a

10% deviance from values calculated using a Rayleigh approximation. Rough estimates of the expected trapping or propulsion forces can therefore be calculated by knowing the electromagnetic intensity distribution in a slot waveguide and the optical constants of the system.

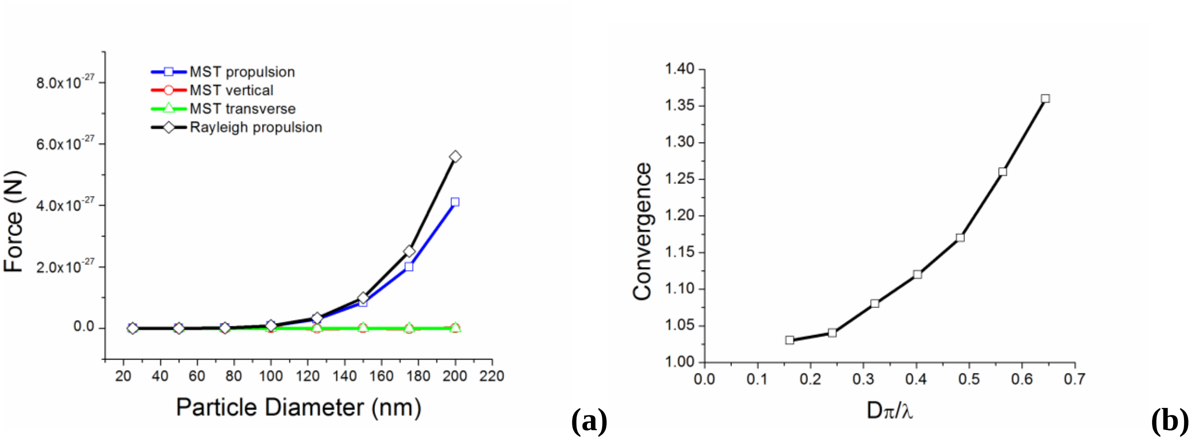

Figure 1: Comparison of Maxwell Stress Tensor (MST) to Rayleigh scattering. (a) Plot of simulated optical force due to a plane wave incident on a particle and calculated forces using the Rayleigh scattering equation for increasing particle sizes. (b) Plot of convergence ($1 - F_{MST}/F_{Ray}$) versus the Rayleigh criterion ($D\pi/\lambda$).

II. Trapping Stiffness and Stability for Diffraction-Limited Optical Tweezer

We can determine the scattering and gradient forces on nanoparticles using equations derived for Rayleigh particles, which assumes that a particle is much smaller than the wavelength of light incident on it, and the electric field around the particle is relatively uniform. For this case the scattering, adsorption and trapping forces exerted on a particle take the form:

$$F_{scat} = \frac{8\pi^3 I_o \alpha^2 \varepsilon_m^{\ 5/2}}{3c\lambda^4} \tag{1}$$

$$F_{abs} = \frac{2\pi\varepsilon_m I_o}{c\lambda} \operatorname{Im}(\alpha) \tag{2}$$

$$F_{trap} = \frac{n_m \alpha}{2c} \nabla I_o \tag{3}$$

where $\alpha = 3V(\varepsilon\text{-}\varepsilon_m)/(\varepsilon+2\varepsilon_m)$ and is the polarizability of the particle, V is the particle volume, c is the speed of light, ε_p and ε_m are the dielectric constants of the particle and material and I_0 is the optical intensity.

To calculate the relevant forces, requires information on the optical intensity and its gradient. We can determine these by considering a laser beam focused through an optical lens with a numerical aperture of 1.2, which would result in a nearly diffraction limited spot size. We can find the intensity profile, at the axial position where the beam is most focused:

$$I(r) = I_0 \exp\left(\frac{-2r^2}{w_0^2}\right) \tag{4}$$

where I is the optical intensity, I_0 is the peak optical intensity, r is the distance from the center of the beam, and w_0 is the spot size. We can similarly, determine the gradient of intensity, by taking the derivative of equation 4. Using this information and equations 1 through 3, we can determine the force profiles for the optical tweezer. Finally, we can use the equations for stability and trap stiffness (slope of F_{trap} profile) to calculate the values for the optical tweezer.

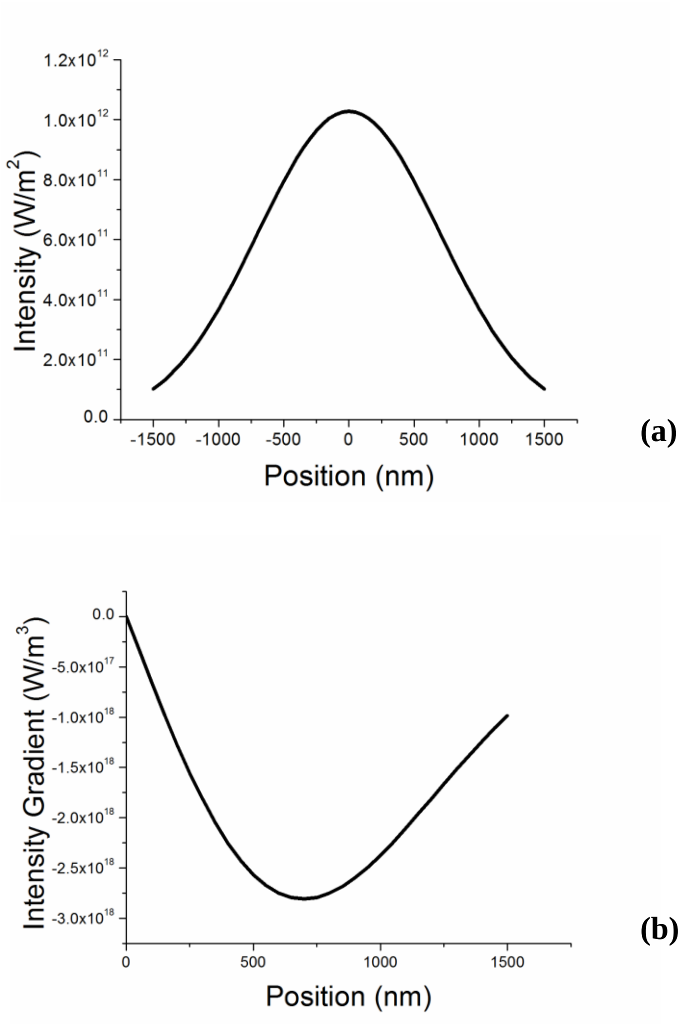

Figure 2: Diffraction limited beam optical intensity profile. Plot of optical intensity versus the radial distance from the center of the beam for a diffraction limited 1550 nm laser spot. (b) Plot of the gradient of optical intensity versus the radial distance from the beam.